OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4761

Mechanics 1

Monday **22 MAY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 6 printed pages and 2 blank pages.

Section A (36 marks)

1 A particle is thrown vertically upwards and returns to its point of projection after 6 seconds. Air resistance is negligible.

Calculate the speed of projection of the particle and also the maximum height it reaches. [4]

2 Force $\mathbf{F}_1$ is $\begin{pmatrix} -6 \\ 13 \end{pmatrix}$N and force $\mathbf{F}_2$ is $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$N, where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are vectors east and north respectively.

(i) Calculate the magnitude of $\mathbf{F}_1$, correct to three significant figures. [2]

(ii) Calculate the direction of the force $\mathbf{F}_1 - \mathbf{F}_2$ as a bearing. [3]

Force $\mathbf{F}_2$ is the resultant of all the forces acting on an object of mass 5 kg.

(iii) Calculate the acceleration of the object and the change in its velocity after 10 seconds. [3]

3 A train consists of an engine of mass 10 000 kg pulling one truck of mass 4000 kg. The coupling between the engine and the truck is light and parallel to the track.

The train is accelerating at 0.25 m s^{-2} along a straight, level track.

(i) What is the resultant force on the train in the direction of its motion? [2]

The driving force of the engine is 4000 N.

(ii) What is the resistance to the motion of the train? [1]

(iii) If the tension in the coupling is 1150 N, what is the resistance to the motion of the truck? [2]

With the same overall resistance to motion, the train now climbs a uniform slope inclined at 3° to the horizontal with the same acceleration of 0.25 m s^{-2}.

(iv) What extra driving force is being applied? [3]

4 Fig. 4 shows the unit vectors **i** and **j** in the directions of the cartesian axes Ox and Oy, respectively. O is the origin of the axes and of position vectors.

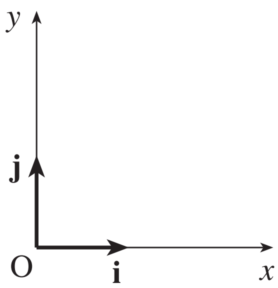

Fig. 4

The position vector of a particle is given by $\mathbf{r} = 3t\mathbf{i} + (18t^2 - 1)\mathbf{j}$ for $t \geqslant 0$, where t is time.

(i) Show that the path of the particle cuts the x-axis just once. [2]

(ii) Find an expression for the velocity of the particle at time t.

Deduce that the particle never travels in the **j** direction. [3]

(iii) Find the cartesian equation of the path of the particle, simplifying your answer. [3]

5 You should neglect air resistance in this question.

A small stone is projected from ground level. The maximum height of the stone above horizontal ground is 22.5 m.

(i) Show that the vertical component of the initial velocity of the stone is $21\ \text{m s}^{-1}$. [2]

The speed of projection is $28\ \text{m s}^{-1}$.

(ii) Find the angle of projection of the stone. [2]

(iii) Find the horizontal range of the stone. [4]

Section B (36 marks)

6 A toy car is travelling in a straight horizontal line.

One model of the motion for $0 \leqslant t \leqslant 8$, where t is the time in seconds, is shown in the velocity–time graph Fig. 6.

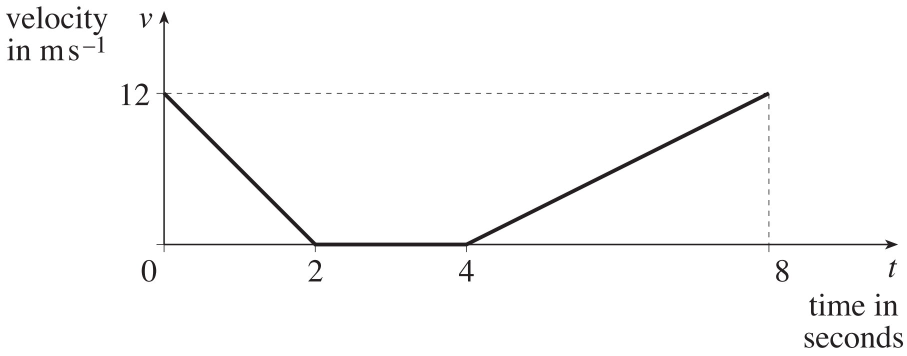

Fig. 6

(i) Calculate the distance travelled by the car from $t = 0$ to $t = 8$. [2]

(ii) How much less time would the car have taken to travel this distance if it had maintained its initial speed throughout? [1]

(iii) What is the acceleration of the car when $t = 1$? [2]

From $t = 8$ to $t = 14$, the car travels 58.5 m with a new constant acceleration, a m s^{-2}.

(iv) Find a. [2]

A second model for the velocity, v m s^{-1}, of the toy car is

$$v = 12 - 10t + \tfrac{9}{4}t^2 - \tfrac{1}{8}t^3, \text{ for } 0 \leqslant t \leqslant 8.$$

This model agrees with the values for v given in Fig. 6 for $t = 0, 2, 4$ and 6. [Note that you are not required to verify this.] Use this second model to answer the following questions.

(v) Calculate the acceleration of the car when $t = 1$. [3]

(vi) Initially the car is at A. Find an expression in terms of t for the displacement of the car from A after the first t seconds of its motion.

Hence find the displacement of the car from A when $t = 8$. [5]

(vii) Explain with a reason what this model predicts for the motion of the car between $t = 2$ and $t = 4$. [3]

7 A box of weight 147 N is held by light strings AB and BC. As shown in Fig. 7.1, AB is inclined at α to the horizontal and is fixed at A; BC is held at C. The box is in equilibrium with BC horizontal and α such that $\sin \alpha = 0.6$ and $\cos \alpha = 0.8$.

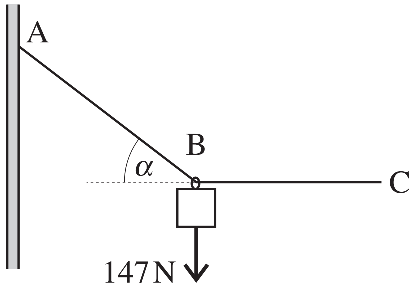

Fig. 7.1

(i) Calculate the tension in string AB. [3]

(ii) Show that the tension in string BC is 196 N. [2]

As shown in Fig. 7.2, a box of weight 90 N is now attached at C and another light string CD is held at D so that the system is in equilibrium with BC still horizontal. CD is inclined at β to the horizontal.

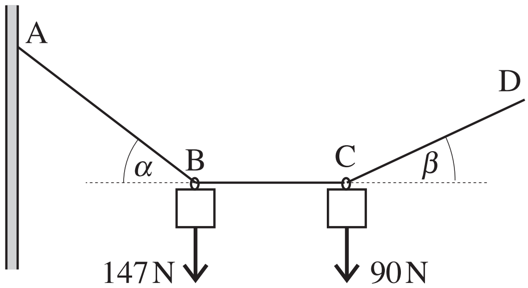

Fig. 7.2

(iii) Explain why the tension in the string BC is still 196 N. [2]

(iv) Draw a diagram showing the forces acting on the box at C.

Find the angle β and show that the tension in CD is 216 N, correct to three significant figures. [7]

The string section CD is now taken over a smooth pulley and attached to a block of mass M kg on a rough slope inclined at 40° to the horizontal. As shown in Fig. 7.3, the part of the string attached to the box is still at β to the horizontal and the part attached to the block is parallel to the slope. The system is in equilibrium with a frictional force of 20 N acting on the block **up** the slope.

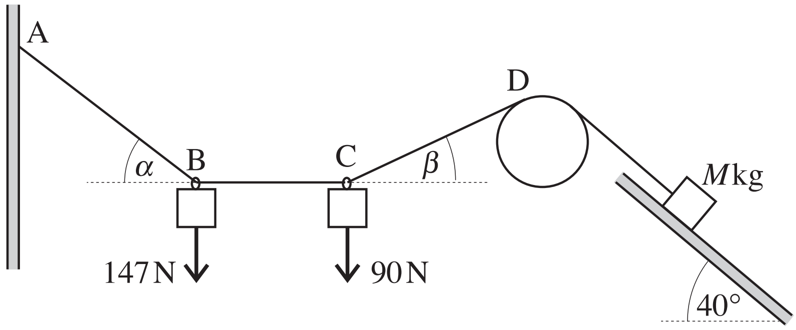

Fig. 7.3

(v) Calculate the value of M. [4]